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4.1/43)

$f(x) = (2x)^x$ on $[0.1, 1]$

so

$(\ln f(x)) = \ln (2x)^x = x \ln(2x)$

taking deriv

$\frac{1}{f(x)} \cdot f'(x) = \left(\ln(2x) + x \frac{1}{2x}(2) \right)$

$\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$

$f'(x) = f(x)\left(\ln(2x) + 1 \right)$

$f'(x) = (2x)^x \left(\ln(2x) + 1 \right)$

$= (2x)^x \left(\ln 2 + \ln x + 1 \right)$

$\ln(2x)$

$\Downarrow$

$\frac{1}{(2x)} \frac{d}{dx}(2x) = \frac{1}{2x}(2) = \frac{2}{2x} = \frac{1}{x}$

$\frac{d}{dx}(\ln 2 + \ln x)$

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83a) If $f'(x) > 0$ and $f''(x) < 0$ on an interval

f increasing ✓ and $f'(x)$ is decreasing ↓

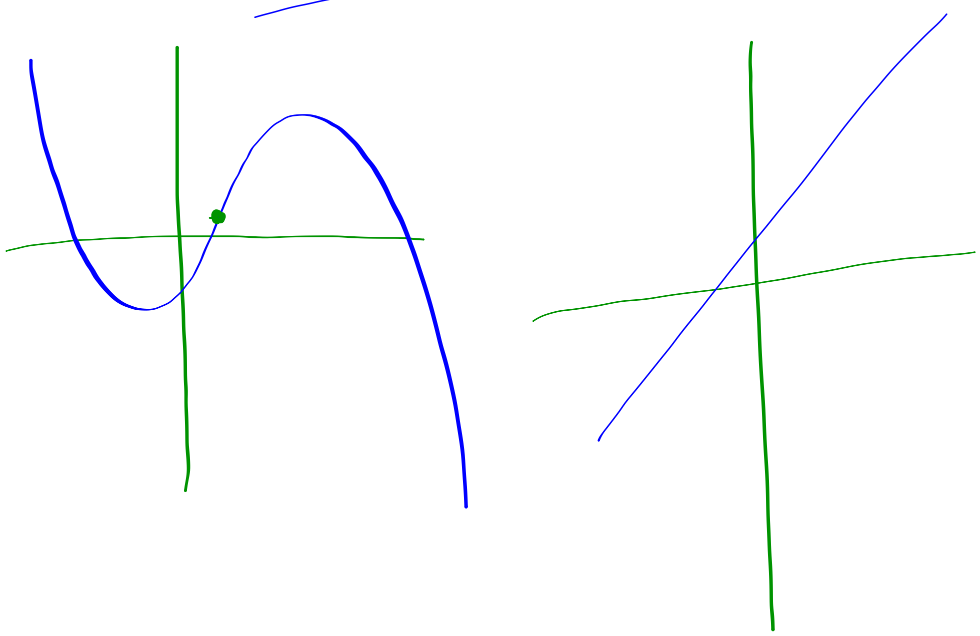
Then f is increasing at a decreasing rate

T

b) If $f'(c) > 0$ and $f''(c) = 0$ then

f(x) increasing

f has a local maximum at c



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c) Two functions that differ by an additive constant both increase and decrease on the same intervals.

Let $y = f(x)$

$Y = f(x) + C$

$y' = f'(x)$

$Y' = f'(x) + 0$

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83d) If f and g increase on an interval, then the product fg also increases

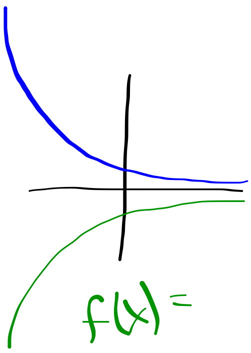
f increasing
$f'(x) > 0$

g increasing
$g'(x) < 0$

$\frac{d}{dx}(fg) =$

$f'g + fg'$

$+ \quad - \quad - \quad +$



$f(x) = -e^{-x}$

$f \cdot f = -e^{-x} \cdot (-e^{-x})$
$= +e^{-2x}$

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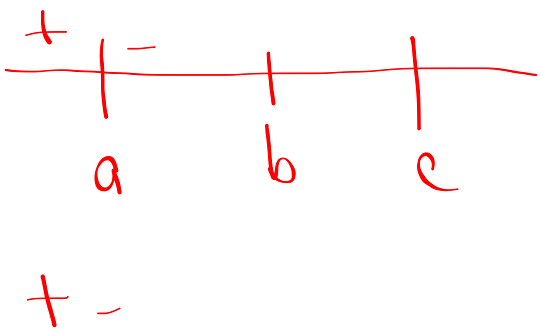
83e There exists a function f that
($\exists$) is continuous on $(-\infty, \infty)$
with exactly 3 critical points,
all of which correspond to local
maxima.



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4.1/68) $f(x) = 6x^4 - 16x^3 - 45x^2 + 54x + 23$ day 58 $[-5, 5]$

crit pt a $f'(x) = 24x^3 - 48x^2 - 90x + 54$

$= 6(4x^3 - 8x^2 - 15x + 9)$

$\Rightarrow f'(x) = 0 \Rightarrow 4x^3 - 8x^2 - 15x + 9 = 0$

b use gf

$\Rightarrow (x-3)(2x-1)(2x+3) = 0$

abs min

$$\begin{array}{r} 4x^2 + 4x - 3 \\ x - 3 \overline{)\, 4x^3 - 8x^2 - 15x + 9} \\ -(4x^3 - 12x^2) \\ \hline 4x^2 - 15x + 9 \\ -(4x^2 - 12x) \\ \hline -3x + 9 \end{array}$$

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